

PAPER

# Fault-Tolerant and Self-Stabilizing Protocols Using an Unreliable Failure Detector

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**SUMMARY** We investigate possibility of fault-tolerant and self-stabilizing protocols (*fts*s protocols) using an unreliable failure detector. Our main contribution is (1) to newly introduce *k-accuracy* of an unreliable failure detector, (2) to show that *k-accuracy* of a failure detector is necessary for any *fts*s *k*-group consensus protocol, and (3) to present three *fts*s *k*-group consensus protocols using a *k*-accurate and weakly complete failure detector under the read/write daemon on complete networks and on  $(n - k + 1)$ -connected networks, and under the central daemon on complete networks.

**key words:** distributed algorithms, self-stabilization, fault-tolerance, failure detector, *x*-group consensus

## 1. Introduction

Research on protocols that are both *fault-tolerant* and *self-stabilizing* is important to develop *truely* reliable distributed systems. A *self-stabilizing* protocol is a protocol that eventually achieves its intended behavior regardless of the initial network configuration. A self-stabilizing protocol tolerates any number of and any kind of *transient* faults in a sense that it can converge from any configuration resulted by transient faults if no further fault occurs for a sufficiently long period of time. On the other hand, a *t-fault-tolerant* protocol (for a *specific permanent* fault model) is a protocol that always achieves its intended behavior from a designated initial configuration regardless of at most *t* faults.

Gopal and Perry [1] first combined the concepts of fault-tolerance and self-stabilization. They consider the *general omission faults* (i.e., send and/or receive omission, and/or crashing), and presented a compiler that transforms a fault-tolerant protocol into a fault-tolerant and self-stabilizing protocol for a synchronous system. They also showed a fault-tolerant and self-stabilizing consensus protocol using unreliable failure detectors [2], [3] on asynchronous systems. Anagnostou and Hadzilacos [4] considered the crash faults. They defined a class of problems called *failure-sensitive* problems that includes the *counting* problem and the *leader election*, and showed that no 1-fault-tolerant and self-stabilizing protocol exists for the *failure-sensitive* prob-

lems. They also presented randomized 1-fault-tolerant and self-stabilizing protocol for the unique naming problem on ring networks. Masuzawa [5] defined the topology problem as a generalized problem of the counting problem. He considered the crash faults and presented a  $(c - 1)$ -fault-tolerant and self-stabilizing protocol for the topology problem on *c*-connected networks under the assumption that each processor knows the neighbors' identifiers. He also showed that there exists no 1-fault-tolerant and self-stabilizing protocol using only either the neighbors' identifiers or the knowledge of connectivity. Beauquier and Kekkonen-Moneta [6] considered the crash fault and tried to clarify the problems for which there exist *k*-fault-tolerant and self-stabilizing protocols. They also presented 1-fault-tolerant and self-stabilizing protocols for some problems on ring networks.

In this paper, we consider the crash faults, and investigate possibility of fault-tolerant and self-stabilizing protocols using a failure detector. We extend an accuracy property of a failure detector and newly define a *k-accuracy* property, which guarantees that at least *k* correct processors are never suspected by any processors. We also define the *x-group consensus* problem, which requires correct processors to select common *x* correct processors. This problem is failure sensitive, and a generalized problem of the election problem. Our main results are (1) to show that *k-accuracy* of the failure detector is necessary for a fault-tolerant and self-stabilizing *k*-group consensus protocol, and (2) to present three  $(n - k)$ -fault-tolerant and self-stabilizing *k*-group consensus protocols which use a *k*-accurate and weakly complete failure detector; a space-unbounded protocol on complete networks under the read/write daemon, a space-unbounded protocol on  $(n - k + 1)$ -connected networks under the read/write daemon, and a space-bounded protocol on complete networks under the central daemon, where *n* is the number of processors. Our protocols are based on the *checking and correction* technique, which is widely studied to transform protocols into self-stabilizing ones [7]–[9].

We treat two types of daemons, the read/write daemon and the central daemon. The two types of daemons are different in atomicity of an action of a processor: the read/write daemon assumes finer grain of atomicity. To classify influence of the difference on

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the possibility of self-stabilization is interesting and has been investigated [10]. It is known that there exists a problem that is solvable under the central daemon but is unsolvable under the read/write daemon by self-stabilizing protocols [11], [12]. We have interest with relationship between such atomicity and *fault-tolerant and self-stabilizing* protocols. In this paper, we present only space-unbounded protocols under the read/write daemon, while we can present a space-bounded protocol under the central daemon.

Chandra et al. [2], [3] investigated what information about failures is necessary and sufficient for *fault tolerant* protocols to solve the consensus problem. They showed the weakest (i.e., necessary and sufficient) failure detector for *fault tolerant* consensus protocols. In this paper, we investigate what information about failures is necessary and sufficient for *fault-tolerant and self-stabilizing* protocols to solve the  $k$ -group consensus problem which is a generalized problem of the leader election. In short, our results show that  $k$ -accuracy is necessary and sufficient for *fault-tolerant and self-stabilizing*  $k$ -group consensus protocols.

We remark a “ $\Gamma$ -accurate” failure detector introduced by Guerraoui et al. [13] (independently of our work), where  $\Gamma$  is a subset of processors. The  $\Gamma$ -accuracy is motivated by the observation that processors suspected to be crashed should be restricted when the system is partitioned. Therefore, it specifies a set  $\Gamma$  of processors that are not mistakenly suspected as crashed processors. Our  $k$ -accuracy has a quite different motivation. The  $k$ -accuracy specifies the number of the processors mistakenly suspected to be crashed. Network partitioning is avoided by requiring  $(n - k + 1)$ -connectivity in this paper.

The rest of this paper is organized as follows. Section 2 and Sect. 3 present the computation model and several definitions. Section 4 shows the necessity of the  $k$ -accuracy of the failure detector for fault-tolerant and self-stabilizing  $k$ -group consensus protocols. Three  $(n - k)$ -fault-tolerant and self-stabilizing  $k$ -group consensus protocols are presented in Sect. 5.

## 2. Preliminaries

### 2.1 Model

A network  $N = (P, L)$  consists of a set  $P = \{p_1, p_2, \dots, p_n\}$  of processors and a set  $L$  of communication links (simply called links), where each link is a pair of distinct two processors. If  $(p_i, p_j) \in L$  holds, then  $p_i$  and  $p_j$  are called *neighbors*. A processor is a state machine. Each processor  $p_i$  has a unique identifier  $id_i$ , drawn from some totally ordered set. We adopt the *link-register* model introduced in [14]. Two neighbors  $p_i$  and  $p_j$  communicate using two *shared communication registers* (simply called registers)  $R_{i,j}$  and  $R_{j,i}$ . The register  $R_{i,j}$  can be written only by  $p_i$  and read

only by  $p_j$ . The register  $R_{i,j}$  is called an *output* register of  $p_i$  and an *input* register of  $p_j$ .

A *configuration* of a network is a vector of processor states and register contents. Let  $m$  be the number of the registers, and let  $S_i$  be the set of states of processor  $p_i$  and  $\Sigma_j$  be the set of symbols that can be stored in the  $j^{\text{th}}$  register<sup>†</sup>. The set  $C$  of all possible configurations is

$$C = S_1 \times S_2 \times \dots \times S_n \times \Sigma_1 \times \Sigma_2 \times \dots \times \Sigma_m.$$

A protocol is a collection of algorithms, one for each processor. Activity of processors is managed by a *daemon*. Whenever the daemon activates a processor, the processor executes an atomic step of its algorithm. In this paper, we use two types of daemons. The *central* daemon (C daemon, in short) activates one processor at a time, and the atomic step of a processor consists of (1) reading all its input registers, (2) changing its state, and (3) writing all its output registers. The *read/write* daemon (R/W daemon, in short) activates one processor at a time, and the atomic step of a processor consists of (1) either reading one of its input registers or writing one of its output registers (but not both), and (2) changing its state.

An execution  $E = c_0, c_1, c_2 \dots$  of a protocol  $A$  is an infinite sequence of configurations, where each  $c_{h+1}$  ( $h \geq 0$ ) is reachable from  $c_h$  by a single atomic step of some processor according to  $A$ . Configuration  $c_0$  is called an *initial configuration* of  $E$ . We assume, for each  $h \geq 0$ , an atomic step  $a$  which changes the configuration from  $c_h$  to  $c_{h+1}$  is uniquely determined. That is, the execution implicitly defines a sequence of atomic steps. Note that any non-empty suffix of any execution is also an execution.

A processor is *faulty* if it does not follow the protocol. We consider only *crash faults* of processors: a faulty processor stops prematurely and does nothing from that point on, however, it behaves correctly before stopping. In the model of the state machine, occurrence of the crash fault is modeled as execution of a special step called a *crash step*. The crash step changes the processor state into a special state, *crash state*, and has no effect on registers. In the crash state, no further step can be executed. The crash step can be executed at any state except for the crash state.

Given an execution  $E$  of a protocol  $A$ , let  $\mathcal{F}(E)$  denote a set of faulty processors (i.e. those in the crash state after some point) and  $\mathcal{C}(E)$  ( $= P - \mathcal{F}(E)$ ) denote a set of correct processors. If every processor in  $\mathcal{C}(E)$  makes infinitely many steps in  $E$ , then  $E$  is called a *fair* execution. We consider only a fair execution in this paper, and simply use the term an *execution* for a fair execution. We assume that a network is asynchronous: there is no assumption on the number of

<sup>†</sup>For convenience, we assume a total order on the registers. This order is used only to describe the configuration, and cannot be used in designing protocols.

steps each processor executes in any prefix of an execution. Note that processor faults cannot be detected in such an asynchronous network since it is impossible to determine whether a processor has actually crashed or is only “very slow”.

A *problem* specifies the required behavior of processors. Formally we define a problem to be a set of *legal executions*, which are executions satisfying the problem requirement. A problem  $\Pi$  on a network  $N = (P, L)$  with a set  $F \subseteq P$  of faulty processors is defined by a set of legal executions denoted by  $L_\Pi(N, F)$ . Let  ${}_sL_\Pi(N, F)$  denote a set of all non-empty suffixes of legal executions in  $L_\Pi(N, F)$ .

A *t*-fault-tolerant (*t-ft*) protocol for a problem  $\Pi$  is a protocol whose any execution starting from a designated initial configuration is legal for  $\Pi$  regardless of at most *t* faulty processors. The designated initial configuration is a configuration in which each processor is in a prescribed initial state and each register contains a prescribed symbol as its initial value.

**Definition 1:** Let  $\mathcal{N}$  be a family of networks, *t* be a non-negative integer, and  $\Pi$  be a problem. A protocol *A* is a *t*-fault-tolerant (*t-ft*) protocol for  $\Pi$  in  $\mathcal{N}$ , if any execution *E* of *A* such that *E* starts from the designated initial configuration in any network  $N (\in \mathcal{N})$  and satisfies  $|\mathcal{F}(E)| \leq t$  is in  $L_\Pi(N, \mathcal{F}(E))$ .

A *t*-fault-tolerant and *self-stabilizing* (*t-ftss*) protocol for a problem  $\Pi$  is a protocol such that its any execution *E* converges to some legal execution *L* of  $\Pi$  (i.e., *E* and *L* have a common suffix) regardless of its initial configuration and at most *t* faulty processors.

**Definition 2:** Let  $\mathcal{N}$  be a family of networks, *t* be a non-negative integer, and  $\Pi$  be a problem. A protocol *A* is a *t*-fault-tolerant and *self-stabilizing* (*t-ftss*) protocol for  $\Pi$  in  $\mathcal{N}$ , if any execution *E* of *A* such that *E* starts from any configuration in any network  $N (\in \mathcal{N})$  and satisfies  $|\mathcal{F}(E)| \leq t$  has a suffix *E'* in  ${}_sL_\Pi(N, \mathcal{F}(E))$ .

Usually, the above definition of stabilization is called *pseudo-stabilization* [15], and stabilization is defined by reachability to some legitimate configuration and closure of a set of the legitimate configurations. However, we consider the crash faults that can occur out of control of the protocol, and deal with a *failure-sensitive* problem that changes the legal executions by occurrence of faults. Therefore, it is impossible to design a protocol which guarantees the closure of a set of the legitimate configurations, and we adopt the definition of the pseudo-stabilization.

In this paper, we make some additional assumption on a network. First, we assume that each processor initially knows the identifiers of its neighbors as well as its own identifier and this knowledge cannot be corrupted by transient faults. That is, every processor knows accurate identifiers of itself and its neighbors at any configuration. In the case of complete networks,

this means that every processor initially knows accurate identifiers of all processors.

## 2.2 *x*-Group Consensus

Anagnostou and Hadzilacos [4] showed that *failure-sensitive* problems, including the leader election problem, has no 1-*ftss* protocol. In this paper, we define and consider the *x*-group consensus problem as a generalized problem of the leader election problem, where *x* is a positive integer. The *x*-group consensus problem requires that correct processors select common *x* correct processors. This problem is very attractive since it can be available such an universal solution that first we select *x* correct processors and then the selected *x* processors cooperatively solve a given problem. The *x*-group consensus problem is failure-sensitive, and there exists no 1-*ftss* *x*-group consensus protocol.

**Definition 3** (*x*-group consensus problem): Let  $N = (P, L)$  be a network. Assume that each processor  $p_i$  has a variable  $Active_i$  representing a set of processor identifiers<sup>†</sup>. An execution  $E = c_0, c_1, \dots$  is legal for *x*-group consensus problem  $\Pi$  (i.e.,  $E \in L_\Pi(N, \mathcal{F}(E))$ ), iff there exist a set  $P' (\subseteq \mathcal{C}(E))$  of *x* correct processors (i.e.,  $|P'| = x$ ) and an integer  $h_0$  ( $h_0 \geq 0$ ) such that, in any configuration  $c_h$  ( $h \geq h_0$ ),  $Active_i = \{id_j | p_j \in P'\}$  holds for any correct processors  $p_i (\in \mathcal{C}(E))$ .

## 2.3 Failure Detector

We use an *unreliable failure detector* introduced by Chandra and Toueg [2]. The failure detector consists of a collection of failure detecting processes, one for each processor. The failure detecting process for a processor  $p_i$  repeatedly suspects faulty processors except for  $p_i$  and manages  $p_i$ 's local variable  $FP_i$  representing an identifier set of suspected processors. The change of the value of  $FP_i$  can be modeled by a change of the state of  $p_i$ . For an execution  $E = c_0, c_1, \dots$ , let  $FP_i^{E,h}$  denote a value of  $FP_i$  in configuration  $c_h$ . If  $id_j \in FP_i^{E,h}$  holds, we say that  $p_i$  suspects  $p_j$  in  $c_h$ .

A failure detector is specified by two properties, *completeness* and *accuracy*. Chandra and Toueg [2] considered two completeness properties and four accuracy properties. Strong (resp. weak) completeness guarantees that every faulty processor is eventually suspected by *all* (resp. *some*) correct processors.

**Definition 4** (strong completeness): A failure detector is *strongly complete* if, for any execution *E*, there exists some  $h_0$  such that, for any correct processor  $p_i (\in \mathcal{C}(E))$  and any  $h (\geq h_0)$ ,  $\mathcal{F}(E) \subseteq FP_i^{E,h}$  holds.

**Definition 5** (weak completeness): A failure detector is *weakly complete* if, for any execution *E* and any faulty

<sup>†</sup>For convenience, we use variables and a program to represent a processor state and a state transition function.

processor  $p_i$  ( $\in \mathcal{F}(E)$ ), there exist some correct processor  $p_j$  ( $\in \mathcal{C}(E)$ ) and some  $h_0$  such that, for any  $h$  ( $\geq h_0$ ),  $p_i \in FP_j^{E,h}$  holds.

Accuracy restricts the mistakes of a failure detector. In [3], four accuracy properties, *strong accuracy*, *weak accuracy*, *eventually strong accuracy* and *eventually weak accuracy* are defined. Intuitively, strong accuracy guarantees that no processor is suspected before it crashes, and weak accuracy guarantees that some correct processes is never suspected. Eventually strong (resp. weak) accuracy means that strong (resp. weak) accuracy holds *eventually*. In this paper, we consider some hierarchy between strong and weak accuracy. We newly define *k-accuracy*, which guarantees that at least  $k$  correct processors are not suspected by any processors. Clearly, 1-accuracy is equivalent to the weak accuracy, and any  $k$ -accuracy is weaker than the strong accuracy since it guarantees that any correct processors is never suspected.

**Definition 6** (*k-accuracy*): A failure detector is *k-accurate* if the following holds: for any execution  $E$  satisfying  $|\mathcal{C}(E)| \geq k$ , there exists a set  $P'$  ( $\subseteq \mathcal{C}(E)$ ) of  $k$  correct processors such that, for any processor  $p_i$  and any integer  $h$  ( $\geq 0$ ),  $P' \cap FP_i^{E,h} = \emptyset$  holds.

We can also consider *eventually k-accuracy* property, which means that the  $k$ -accuracy holds *eventually*. However, we does not consider such a property, since self-stabilizing protocols are required to *eventually* achieve their intended behavior, therefore, if we consider an execution only after the  $k$ -accuracy holds, it can be considered that the eventually  $k$ -accuracy is equivalent with the  $k$ -accuracy.

### 3. Necessity of $k$ -Accuracy for $k$ -Group Consensus

In this section, we show that the  $k$ -accuracy of the failure detector is necessary for 1-*ftss*  $k$ -group consensus protocols. We show that there is no 1-*ftss*  $k$ -group consensus protocol for the  $k$ -group consensus problem which uses a  $(k - 1)$ -accurate and strongly complete failure detector. Since a failure detector is specified by completeness and accuracy, and strong completeness is the strongest with respect to completeness, this result implies that  $k$ -accuracy of the failure detector is necessary for 1-*ftss*  $k$ -group consensus protocols, and hence, for any *ftss*  $k$ -group consensus protocols.

First, we define some notations. Let  $c$  and  $c'$  be configurations. Let  $c \overset{i}{\bowtie} c'$  denote a configuration that is identical to  $c$  except that  $p_i$ 's state is the same as in  $c'$ . Let  $ID$  be a set of identifiers. A configuration  $c$  is *ID-consensus* if  $Active_i = ID$  for every processor  $p_i$  which is not in the crash state. Let  $C_k$  denote a set of all *ID-consensus* configurations such that the size of  $ID$  is  $k$ .

**Theorem 1:** There exists no 1-*ftss* protocol for the  $k$ -group consensus problem under the C daemon, even if it can use a  $(k - 1)$ -accurate and strongly complete failure detector.

**(Proof)** Assume that a protocol  $A$  is a 1-*ftss* protocol for the  $k$ -group consensus problem using a  $(k - 1)$ -accurate and strongly complete failure detector in some network family. We consider the following execution  $E = c_0, c_1, \dots$  where  $\mathcal{F}(E) = \emptyset$  and every processor suspects all the processors except for some  $k - 1$  processors and itself. Let  $FP$  be a set of  $n - k + 1$  identifiers such that  $FP_i^{E,h} = FP - \{id_i\}$  for any  $p_i \in P$  and any  $h$  ( $\geq 0$ ).

First, the daemon activates all processors until some *ID-consensus* configuration  $c_{h_1}$  in  $C_k$  is reached. Since  $|ID| = k$  and  $|FP| = n - k + 1$ , there exists some  $id_i$  such that  $id_i \in ID \cap FP$ . We temporarily assume that  $p_i$  in the crash state  $c_{h_1}$ . Since  $A$  is a 1-*ftss* protocol, the daemon can lead the network to some *ID'*-consensus configuration  $c'_{h_2}$  in  $C_k$  where  $id_i \notin ID'$ . The processor  $p_i$  does nothing from  $c_{h_1}$  to  $c'_{h_2}$ , and the other processors cannot distinguish whether  $p_i$  has crashed or is just slow. Therefore, steps from  $c_{h_1}$  to  $c'_{h_2}$  are possible to occur if  $p_i$  is actually correct.

Now consider the case where  $p_i$  is a correct processor. In this case the daemon can leads the network to the configuration  $c_{h_2} = c'_{h_2} \overset{i}{\bowtie} c_{h_1}$  by activating the processor except for  $p_i$ . In  $c_{h_2}$ ,  $Active_i = ID$  and  $Active_j = ID' \neq ID$  for any  $j$  ( $j \neq i$ ), therefore,  $c_{h_2} \notin C_k$ . Since  $A$  is 1-*ftss* protocol, some configuration  $c_{h_3}$  in  $C_k$  is reached again.

By repeating the above strategy, the daemon can schedule processor steps so that configurations not in  $C_k$  appear infinitely often in  $E$ . However,  $A$  is 1-*ftss* protocol, and therefore,  $E$  has a suffix consisting of only configurations in  $C_k$ . A contradiction occurs.  $\square$

Since the R/W daemon has smaller atomicity than C daemon, the R/W daemon can activate processors in the same way as the C daemon. Thus, impossibility results for the C daemon holds for the R/W daemon, and the following corollary holds.

**Corollary 1:** There exists no 1-*ftss* protocol for the  $k$ -group consensus problem under the R/W daemon, even if it can use a  $(k - 1)$ -accurate and strongly complete failure detector.

## 4. $(n - k)$ -*ftss* $k$ -Group Consensus Protocols

### 4.1 Overview

We present the following three  $(n - k)$ -*ftss*  $k$ -group consensus protocols.

- *RWKP* : a space-unbounded protocol under the R/W daemon in complete networks.

- *RWKP'* : a space-unbounded protocol under the R/W daemon in  $(n - k + 1)$ -connected networks.
- *CKP* : a space-bounded protocol under the C daemon in complete networks.

First, we describe the common overview to all protocols. In the description of the protocols in Fig. 1, Fig. 2 and Fig. 3,  $read_{i,j}(x_i)$  denotes that  $p_i$  reads its input register  $R_{j,i}$  and stores the value to its local variable  $x_i$ , and  $write_{i,j}(x_i)$  denotes that  $p_i$  writes the value of its local variable  $x_i$  to its output register  $R_{i,j}$ . If the variable  $x_i$  is partitioned into some fields  $x_i.a$ ,  $x_i.b, \dots$ , we refer the corresponding fields of  $R_{i,j}$  as  $R_{i,j}.a$ ,  $R_{i,j}.b, \dots$ . Let  $S$  be an identifier set. Let  $pick_k(S)$  denote a function returning the smallest  $k$  identifiers in

```

var
  sus_i, Active_i: set of processor ids;
  rsus(1 ≤ j ≤ n): set of processor ids;
begin
  sus_i := ∅; /* initialization */
  repeat forever do
    sus_i := sus_i ∪ FP_i;
    for each j (1 ≤ j ≤ n, j ≠ i) do
      read_{i,j}(rsus);
      sus_i := sus_i ∪ rsus;
    Active_i := pick_k({id_1, ..., id_n} - sus_i);
    for each j (1 ≤ j ≤ n, j ≠ i) do
      write_{i,j}(sus_i);
  end

```

**Fig. 1**  $(n - k)$ -ft protocol *KP*: code for  $p_i$ .

```

var
  sus_i, Active_i, sdata.sus, rdata.sus
    : set of processor ids;
  sdata.vn, rdata.vn : integer;
begin
  repeat forever do
    sus_i := sus_i ∪ FP_i;
    for each j (1 ≤ j ≤ n, j ≠ i) do
      read_{i,j}(rdata);
    if rdata.vn = vn_i then
      sus_i := sus_i ∪ rdata.sus;
    if |sus_i| > n - k then
      sus_i := ∅; vn_i := vn_i + 1;
    else
      Active_i := pick_k({id_1, ..., id_n} - sus_i);
    else if rdata.vn > vn_i then
      vn_i := rdata.vn; sus_i := rdata.sus;
  sdata.sus := sus_i; sdata.vn := vn_i;
  for each j (1 ≤ j ≤ n, j ≠ i) do
    write_{i,j}(sdata)
  end

```

**Fig. 2**  $(n - k)$ -ftss protocol *RWKP*: code for  $p_i$ .

$S$ . Note that a variable  $FP_i$  denote an identifier set of suspected processors and it is under the control of the failure detecting process for  $p_i$ . In this subsection, we present  $(n - k)$ -ftss protocols, and therefore, we consider only such an execution  $E$  that  $\mathcal{F}(e) \leq n - k$  holds.

Our three protocols are based on a  $(n - k)$ -ft protocol *KP* in complete networks under the R/W daemon (Fig. 1). The protocol *KP* uses a  $k$ -accurate and weakly complete failure detector. The protocol is correct if all  $Active_i (= pick_k(\{id_1, \dots, id_n\} - sus_i))$  of

```

var
  sus_i, sdata_{i,j}.sus, rdata_{j,i}.sus, Active_i, Rcv_i
    : set of processor ids;
  sdata_{i,j}.F, rdata_{j,i}.F: boolean;
  /* flag for communication mechanism */
  sdata_{i,j}.R, rdata_{j,i}.R: boolean; /* reset request */
  mode_i: NORMAL or RESET;
begin
  repeat forever do
    /* receive messages */
    Rcv_i := ∅;
    for each j (j ≠ i) do
      read_{i,j}(rdata_{j,i});
      /* select newly received messages */
      if first_read(R_{i,j}) = true
        then Rcv_i := Rcv_i ∪ {j};
    /* update sus_i */
    if there exists j ∈ Rcv_i s.t. rdata_{j,i}.R = true
      then /* case: reset-request */
        sus_i := ∅; mode_i := NORMAL;
      else /* case: normal message */
        /* update a total suspicion */
        sus_i := sus_i ∪ ∪_{j ∈ Rcv_i} rdata_{j,i}.sus ∪ FP_i;
        if |sus_i| > n - k
          then /* inconsistency is detected */
            /* reset itself */
            sus_i := ∅; mode_i := RESET;
          else
            /* select k correct processors */
            Active_i := pick_k(\{id_1, ..., id_n\} - sus_i);
            mode_i := NORMAL;
        /* set messages for processors read previous messages. */
        for each j ∈ Rcv_i do
          sdata_{i,j}.sus := sus_i; sdata_{i,j}.R := false;
          /* for communication mechanism */
          sdata_{i,j}.F := (sdata_{i,j}.F + 1) mod 2;
        /* update messages if reset mode */
        if mode_i = RESET /* reset mode */
          then
            /* set reset-requests for all processors */
            for each j (j ≠ i) do
              sdata_{i,j}.R := true; /* reset-request */
        /* send messages */
        for each j (j ≠ i) do
          write_{i,j}(sdata_{i,j});
  end.

```

**Fig. 3**  $(n - k)$ -ftss protocol *CKP*: code for  $p_i$ .

correct processors converge to the same set of  $k$  correct processors. To show this convergence, we prove the convergence of a variable  $sus_i$ . A variable  $sus_i$  represents a set of processor identifiers which  $p_i$  itself suspects or  $p_i$  knows some processor suspects. We call  $sus_i$  a *total suspicion* of  $p_i$ . We can observe that every total suspicion monotonically increases with respect to the inclusion relation ' $\subseteq$ ', any correct processor  $p_i$ 's total suspicion will be included by any other correct processor  $p_j$ 's total suspicion after sufficient number of steps, and a total suspicion of every correct processor is bounded from above by a set of all identifiers. Therefore, all total suspicions of correct processors converge to the same set  $sus$  of identifiers. From the  $k$ -accuracy and the weak completeness of the failure detector, this  $sus$  includes all faulty processors and never includes at least  $k$  correct processors (i.e., at most  $n - k$  faulty processors). Therefore, all  $Active_i$  of correct processors converge to the same set of  $k$  correct processors. This means that  $KP$  is an  $(n - k)$ -ft  $k$ -group consensus protocol.

Now, we extend  $(n - k)$ -ft protocol  $KP$  to  $(n - k)$ -ftss protocols. For ftss protocols, we can assume nothing on the initial total suspicion. If some  $sus_i$  initially includes many correct processor identifiers, the size of  $sus_i$  may exceed  $n - k$ . This is inconsistent with the  $k$ -accuracy of the failure detector. In our ftss protocols, each  $p_i$  checks such *inconsistency* (i.e.,  $|sus_i| > n - k$ ) whenever it updates the value of  $sus_i$ . If  $p_i$  detects the inconsistency,  $p_i$  resets its state (sets its total suspicion empty) and attempts to reset a network configuration (set the network to some configuration reachable from the designated initial configuration of  $KP$ ). If the network configuration can be reset, the  $k$ -group consensus problem can be solved.

Note that the protocol  $KP$  has infinite iterations and any processors does not know when the variable  $Active_i$  converges. This is natural because the convergence period of  $Active_i$  depends on both time when processors crash and suspicions of failure detectors.

#### 4.2 Protocols under the R/W Daemon

We present an  $(n - k)$ -ftss protocols for the  $k$ -group consensus problem using a  $k$ -accurate and weakly complete failure detector under the R/W daemon. First, we present a protocol  $RWKP$  (Fig. 2) in complete networks, and then extend it to be applicable to  $(n - k + 1)$ -connected networks.

In  $RWKP$ , each processor  $p_i$  uses a variable  $vn_i$  representing the version number of its local suspicion  $sus_i$ . Each processor exchanges the total suspicion with other processors. When  $p_i$  reads the total suspicion  $rdata.sus$  with version number  $rdata.vn$  from its input register,  $p_i$  updates its total suspicion as follows: (1) if  $rdata.vn > vn_i$ ,  $p_i$  resets its total suspicion and sets  $sus_i$  to  $rdata.sus$ , (2) if  $rdata.vn = vn_i$ ,

$p_i$  adds  $rdata.sus$  to  $sus_i$ , and if it becomes inconsistent, i.e.,  $|sus_i| > n - k$ ,  $p_i$  resets its total suspicion and increments its version number by one, and (3) if  $rdata.vn < vn_i$ ,  $p_i$  ignores  $rdata.sus$  and does nothing.

To prove the correctness of  $RWKP$ , we must show the convergence of the variables  $Active_i$  of all correct processors. This convergence is derived directly from the convergence of the total suspicions of all correct processors. To show this, we use the following lemma. It shows conditions for that all total suspicions converge to the same set which includes all faulty processors. For an execution  $E = c_0, c_1, \dots$ , let  $v^{E,h}$  denote a value of a variable  $v$  in a configuration  $c_h$ .

**Lemma 1:** Consider any protocol in which each processor  $p_i$  has a variable  $sus_i$  of an identifier set and uses a weakly complete failure detector. Let  $E = c_0, c_1, \dots$  be an execution of the protocol. If the following four conditions hold, there exist some set  $sus$  and some  $g_0$  such that  $\mathcal{F}(E) \subseteq sus$  and, for any  $p_i (\in \mathcal{C}(E))$  and any  $g (\geq g_0)$ ,  $sus_i^{E,g} = sus$  holds.

- (1) There exists a set  $S$  of identifiers such that  $sus_i^{E,h} \subseteq S$  holds for any  $p_i (\in \mathcal{C}(E))$  and any  $h$ .
- (2) For any  $p_i (\in \mathcal{C}(E))$  and any  $h$  and  $h'$  ( $h \leq h'$ ),  $sus_i^{E,h} \subseteq sus_i^{E,h'}$  holds.
- (3) For any  $p_i$  and  $p_j$  ( $p_i, p_j \in \mathcal{C}(E)$ ) and any  $h$ , there exists some  $h'$  such that  $sus_i^{E,h} \subseteq sus_j^{E,h'}$ .
- (4) For any  $p_i (\in \mathcal{C}(E))$  and any  $h$  ( $h > 0$ ),  $FP_i^{E,h-1} \subseteq sus_i^{E,h}$  holds.

**(Proof)** For every correct processor, condition (1) means  $sus_i$  has an upper bound (w.r.t. ' $\subseteq$ '), and condition (2) means  $sus_i$  monotonically increases. Therefore, every  $sus_i$  ( $p_i \in \mathcal{C}(E)$ ) converges to some set  $final\_sus_i$ . The condition (3) implies, for any correct processors  $p_i$  and  $p_j$ ,  $final\_sus_i \subseteq final\_sus_j$  and  $final\_sus_j \subseteq final\_sus_i$ , therefore,  $final\_sus_i = final\_sus_j$  holds. Therefore, there exist some set  $sus$  and some  $g_0$  such that, for any  $p_i (\in \mathcal{C}(E))$  and any  $g (\geq g_0)$ ,  $sus_i^{E,g} = sus$  holds. Moreover, by (2) and (4),  $FP_i^{E,h-1} \subseteq final\_sus_i$ , and  $\bigcup_{p_i \in \mathcal{C}(E)} FP_i^{E,h-1} \subseteq sus$  hold for any  $h$  ( $h > 0$ ). By the weak completeness,  $\mathcal{F}(E) \subseteq sus$  holds.  $\square$

Now we show the correctness of  $RWKP$ .

**Lemma 2:** For any execution  $E = c_0, c_1, \dots$  of  $RWKP$  under the R/W daemon, there exists an integer  $vn$  such that  $vn_i^{E,h} \leq vn$  holds for any  $i$  and  $h$ .

**(Proof)** Let  $max$  be the maximum version number appears in  $c_0$ . Assume that the lemma does not hold, i.e., some version numbers have no upper bound. Let  $c_g$  the first configuration in which some version number becomes no smaller than  $max + 2$ . Let  $p_i$  be a processor such that  $vn_i^{E,g} = max' (\geq max + 2)$ . In  $c_{g-1}$ , no version number is more than  $max + 1$ , therefore, in a step from  $c_{g-1}$  to  $c_g$ ,  $p_i$  ought to detect the

inconsistency  $|sus_i| > n - k$  and increment  $vn_i$  from  $max' - 1$  to  $max'$ . Let  $sus'$  denote the value of this inconsistent  $sus_i$ . In *RWKP*, every total suspicion is computed from the total suspicions with the same version number, therefore,  $p_i$  computes  $sus'$  from the total suspicions with version number  $max' - 1$ . Since  $max' - 1 > max$ , every processor with version number  $max' - 1$  has reset its total suspicion at least once. This implies that every total suspicion with version number  $max' - 1$  includes only the identifiers suspected by the failure detector. That is,  $sus' \subseteq \bigcup_{p_i \in P, 0 \leq h < g} FP_i^{E,h}$  holds. However,  $|\bigcup_{p_i \in P, 0 \leq h < g} FP_i^{E,h}| \leq n - k$  holds by the  $k$ -accuracy, a contradiction occurs.  $\square$

**Theorem 2:** If the failure detector is  $k$ -accurate and weakly complete, the protocol *RWKP* is an  $(n - k)$ -ftss  $k$ -group consensus protocol in complete networks under the R/W daemon.

**(Proof)** Consider any execution  $E = c_0, c_1, \dots$  of *RWKP*. By Lemma 2, there exists the maximum value  $max\_vn$  of version numbers of correct processors appear in  $E$ . Let  $c_h$  be a configuration in which  $vn_i^{E,h} = max\_vn$  for some correct processor  $p_i$ . Since each version number never decreases,  $vn_i^{E,h'} = max\_vn$  holds for any  $h' \geq h$ . Fairness of executions guarantees that, for any correct processor  $p_j$ ,  $p_i$  writes  $max\_vn$  to the register  $R_{i,j}$  as a value of  $vn_i$ , and after then  $p_j$  reads  $R_{i,j}$ . After  $p_j$  reads the value  $max\_vn$ ,  $vn_j$  becomes at least  $max\_vn$ . Since  $max\_vn$  is the maximum value of version numbers of correct processors,  $vn_j$  becomes  $max\_vn$  and remains  $max\_vn$  after that. That is,  $E$  has some suffix  $E' = c'_0, c'_1, \dots$  such that  $vn_j^{E',h} = max\_vn$  for any correct processor  $p_j$  and any  $h \geq 0$ .

In  $E'$ , any correct processor never increments its version number, therefore, it never resets its total suspicion. Now we show that Lemma 1 can be applied for  $E'$ . (1) Clearly, every  $sus_i^{E',h} \subseteq \{id_i | p_i \in P\}$  holds for any  $p_i \in \mathcal{C}(E')$  and  $h$ . (2)  $sus_i^{E',h} \subseteq sus_i^{E',h'}$  holds for any  $p_i \in \mathcal{C}(E')$  and any  $h$  and  $h'$  ( $h \leq h'$ ). (3) For any  $p_i$  and  $p_j$  ( $p_i, p_j \in \mathcal{C}(E')$ ) and any  $h$ , there exists some  $\bar{h}$  and  $h'$  ( $h \leq \bar{h} \leq h'$ ) such that  $p_i$  writes  $sus_i^{E',\bar{h}}$  ( $\supseteq sus_i^{E',h}$ ) to  $R_{i,j}$  and then  $p_j$  reads  $sus_i^{E',\bar{h}}$  from  $R_{i,j}$  and sets  $sus_j^{E',h'} (\supseteq sus_i^{E',h})$ . Finally, (4) for any  $sus_i$  ( $p_i \in \mathcal{C}(E')$ ) and any  $h$  ( $h > 0$ )  $FP_i^{E',h-1} \subseteq sus_i^{E',h}$  holds. By the above (1), (2), (3) and (4), and the facts of  $\mathcal{C}(E') = \mathcal{C}(E)$  and  $\mathcal{F}(E') = \mathcal{F}(E)$ , there exist some set  $sus$  and some  $g_0$  such that  $\mathcal{F}(E) \subseteq sus$  and, for any  $p_i \in \mathcal{C}(E)$  and  $g$  ( $g \geq g_0$ ),  $sus_i^{E',g} = sus$  hold. Since any correct processor ( $\in \mathcal{C}(E)$ ) never resets its total suspicion in  $E'$ ,  $|sus| \leq n - k$  holds. Let  $Active$  be the value of  $pick_k(\{id_1, id_2, \dots, id_n\} - sus)$ . Then,  $|Active| = k$  and  $Active \subseteq \{id_j | p_j \in \mathcal{C}(E)\}$  hold. For any  $p_i \in \mathcal{C}(E)$  and any  $g \geq g_0$ ,  $sus_i^{E',g} = sus$  holds, and therefore,

and  $Active_i^{E',g} = Active$  holds. Since  $E'$  is a suffix of  $E$ , this implies that  $E$  is a legal execution for the  $k$ -group consensus problem.  $\square$

The protocol *RWKP* in complete networks can be extended to an  $(n - k)$ -ftss protocol in any  $(n - k + 1)$ -connected networks. Masuzawa [5] proposed an  $(n - k)$ -ftss topology protocol in  $(n - k + 1)$ -connected networks under the assumption that each processor initially knows the identifiers of its neighbors. In an execution of the topology protocol, each processor eventually obtains the network topology including an accurate set of identifiers of all processors. Now consider the composite protocol *RWKP'* of *RWKP* and the topology protocol. In *RWKP'*, each processor alternatively executes a step of *RWKP'* and a step of the topology protocol. The differences between *RWKP* and *RWKP'* are (1) each processor initially knows an accurate set of identifiers of all processors in *RWKP*, and (2) any two processors can directly communicate via the registers between them in *RWKP*. These are resolved as follows. (1) In any execution of *RWKP'*, each processor eventually obtains an accurate set of identifiers of all processors, and (2) any two correct processors have a path between them consisting of only correct processors, and fairness of executions guarantees that every total suspicion is propagated through the path unless it meets with a larger version number. Therefore, *RWKP'* is an  $(n - k)$ -ftss  $k$ -group consensus protocol in  $(n - k + 1)$ -connected networks.

**Corollary 2:** If the failure detector is  $k$ -accurate and weakly complete, the protocol *RWKP'* is an  $(n - k)$ -ftss  $k$ -group consensus protocol in  $(n - k + 1)$ -connected networks under the R/W daemon.

### 4.3 Protocol under the C Daemon

The protocol *RWKP* and *RWKP'* are space-unbounded, since they use an unbounded variable  $vn_i$ . In this subsection, we present a space-bounded ftss  $k$ -group consensus protocol *CKP* (Fig. 3) in complete networks under the C daemon. Note that we cannot obtain a space-bounded protocol on any  $(n - k + 1)$ -connected network by combining the protocol *CKP* and the topology protocol [5], since the topology protocol is space-unbounded.

In *CKP*, when some processor  $p_i$  resets by detection of the inconsistency  $|sus_i| > n - k$ , the other processors reset synchronously. Synchronous reset means that each  $p_j$  ( $\neq p_i$ ) resets in the first step of itself after  $p_i$  resets, and, after that time on, exchanges messages only with reset processors. To implement this synchronous reset, *CKP* provides the following communication mechanism.

- **Detection of the duplicated read by the reader:** When  $p_j$  reads  $R_{i,j}$ ,  $p_j$  can find whether  $p_i$  wrote  $R_{i,j}$  after the last read of  $R_{i,j}$ .

- **Detection of the unread by the writer:** When  $p_i$  reads  $R_{j,i}$ ,  $p_i$  can find whether  $p_j$  read  $R_{i,j}$  after the last write to  $R_{i,j}$ .

These mechanism can be implemented if each of  $p_i$  and  $p_j$  can find which processor executed last in a step of itself. For this purpose, *CKP* uses a flag field  $F$  in each register. When  $p_i$  writes to  $R_{i,j}$ ,  $p_i$  updates  $R_{i,j}.F$  so as to be  $last\_processor(p_i, p_j) = p_i$ , where the function  $last\_processor$  is defined as follows.

$$last\_processor(p_i, p_j) = \begin{cases} p_i & \text{if } (id_i < id_j \wedge R_{i,j}.F \neq R_{j,i}.F) \\ & \vee (id_i > id_j \wedge R_{i,j}.F = R_{j,i}.F) \\ p_j & \text{otherwise} \end{cases}$$

In Fig. 3, the following predicate is used.

$$\begin{aligned} & first\_read(R_{j,i}) \\ & = (id_i < id_j \wedge sdata_{i,j}.F = rdata_{j,i}.F) \\ & \vee (id_i > id_j \wedge sdata_{i,j}.F \neq rdata_{j,i}.F) \end{aligned}$$

Each processor  $p_i$  decides whether  $p_i$  reads  $R_{j,i}$  first after the last write to  $R_{j,i}$  using this predicate.

Synchronous reset is implemented as follows. First, consider the case where some processor  $p_i$  detects the inconsistency  $|sus_i| > n - k$ , resets itself, and then requires all the other processors to reset themselves by setting a reset-request flag  $R$  to *true* in its each output register. Under the C daemon, the detection of the inconsistency and the set of reset-request flags are executed in a single atomic step. We call this atomic step a *reset-request step*. The processors  $p_i$  holds the reset-request flag in  $R_{i,j}$  *true* until the processor  $p_j$  reads this request. On the other hand,  $p_j$  ( $\neq p_i$ ) reads this request in the first step of itself after the reset-request step, and then resets itself.

First, we prove the communication mechanism.

**Lemma 3:** For any execution  $E$  of *CKP* under the C daemon, there exists some suffix of  $E$  in which, for any step  $a_i$  of any  $p_i$ , (1) there exists the last step  $a'_i$  of  $p_i$  before  $a_i$  in  $E$  and, (2) for any  $p_j$  ( $\neq p_i$ ),  $first\_read(R_{j,i})$  holds in  $a_i$  if and only if there exists a step of  $p_j$  between  $a'_i$  and  $a_i$ .

**(Proof)** Let  $E'$  be some suffix of  $E$  after every correct processor executes at least one step and all faulty processors have crashed. Consider any step of any  $p_i$  in  $E'$ . Since only correct processors execute steps in  $E'$ , (1) there exists the last step  $a'_i$  of  $p_i$  before  $a_i$  in  $E$ .

Let  $p_j$  be any processor (maybe a faulty processor). In  $a'_i$ ,  $p_i$  reads  $R_{j,i}$  and stores the value to  $rdata_{j,i}$ . The processor  $p_i$  appends  $j$  to  $Rcv_i$  if and only if  $first\_read(R_{j,i})$  holds. At the end of  $a'_i$ ,  $p_i$  increments  $sdata_{i,j}.F$  if  $j$  is in  $R_{j,i}$ , and then writes  $sdata_{i,j}.F$  to  $R_{i,j}$ . At that time,  $last\_processor(p_i, p_j) = p_i$  holds.

If  $p_j$  executes a step  $a_j$  between  $a'_i$  and  $a_i$ , at the end of  $a_j$ ,  $last\_processor(p_i, p_j) = p_j$  holds, and

it continues to hold until  $a_i$  is executed. On the other hand, if  $p_j$  executes no step between  $a'_i$  and  $a_i$ ,  $last\_processor(p_i, p_j) = p_i$  continues to hold until  $a_i$  is executed.

In  $a_i$ ,  $sdata_{i,j}.F = R_{i,j}.F$  holds since only  $p_i$  writes to  $R_{i,j}$  and  $rdata_{j,i}.F = R_{j,i}.F$  holds since  $p_i$  first reads  $R_{j,i}$  and sets  $rdata_{j,i} = R_{j,i}$ . Therefore,  $last\_processor(p_i, p_j) = p_j$  holds if and only if  $first\_read(R_{j,i})$  holds. This implies that (2)  $first\_read(R_{i,j})$  holds in  $a_i$  if and only if there exists a step of  $p_j$  between  $a'_i$  and  $a_i$ .  $\square$

Next, we prove the synchronous reset.

**Lemma 4:** Any execution  $E$  of *CKP* under the C daemon has some suffix in which no processor executes a reset-request step.

**(Proof)** Consider some suffix  $E'$  satisfying Lemma 3. Let  $p_i$  be the first processor which executes a reset-request step  $a_i$  in  $E'$  if exists. In the reset step  $a_i$ ,  $p_i$  writes the value *true* to each output register  $R_{i,j}.R$ . After  $a_i$ ,  $p_i$  never changes the value of  $R_{i,j}.R$  until  $p_j$  reads it. If a processor  $p_j$  ( $\neq p_i$ ) executes its step after  $a_i$ ,  $p_j$  reads the value *true* from  $R_{i,j}.R$  in the first step  $a_j$  after that reset step, and then resets itself. In this  $a_j$ ,  $p_j$  also reads all its input registers, therefore,  $p_j$  reads, in  $a_j$  at the latest, all messages written before the reset step. In  $a_j$ ,  $p_j$  actually ignores the messages that  $p_j$  read from its input registers. In later steps,  $p_j$  ignores such ignored messages even if  $p_j$  reads them again. Therefore, after the reset step, every processor creates its total suspicion from the messages written after the reset-request step  $a_i$  and the suspected identifiers from its failure detecting process. This implies that, at the end of any step of any  $p_j$  ( $\in P$ ) after the reset step,  $sus_j \subseteq \bigcup_{p_i \in P, 0 \leq h} FP_i^{E,h}$  holds, therefore,  $|sus_j| \leq n - k$  holds from the  $k$ -accuracy of the failure detector and  $p_j$  does not reset itself. That is,  $E$  has some suffix in which no processor executes a reset-request step.  $\square$

Now we show the correctness of *CKP*.

**Theorem 3:** If the failure detector is  $k$ -accurate and weakly complete, the protocol *CKP* is an  $(n - k)$ -ftss  $k$ -group consensus protocol in complete networks under the C daemon.

**(Proof)** We first show the convergence of variables  $sus_i$ . By Lemma 4, for any execution  $E$  of *CKP* under the C daemon, there exists some suffix  $E'$  in which no processor executes a reset-request step. In *CKP*, each processor  $p_i$  ignores any message if  $p_i$  has already read it. Since no processor executes a reset-request step in  $E'$ , each  $p_i$  resets itself at most once when it first reads a reset request. Therefore,  $E'$  has some suffix  $E'' = c''_0, c''_1, \dots$  in which no processor resets itself. Now we show that Lemma 1 can be applied for  $E''$ . (1) Clearly, every  $sus_i^{E'',h} \subseteq \{id_i | p_i \in P\}$

holds for any  $p_i$  and  $h$ . (2)  $sus_i^{E'',h} \subseteq sus_i^{E'',h'}$  holds for any  $p_i \in \mathcal{C}(E'')$  and any  $h$  and  $h'$  ( $h \leq h'$ ). (3) Fairness of executions guarantees that for any  $p_i$  and  $p_j$  ( $p_i, p_j \in \mathcal{C}(E'')$ ) and any  $h$ , there exists some  $h'$  such that  $sus_i^{E'',h} \subseteq sus_j^{E'',h'}$ . Finally, (4) for any  $sus_i$  ( $p_i \in \mathcal{C}(E'')$ ) and any  $h$  ( $h > 0$ ),  $FP_i^{E'',h-1} \subseteq sus_i^{E'',h}$  holds.

By the above (1), (2), (3) and (4), and the facts of  $\mathcal{C}(E'') = \mathcal{C}(E)$  and  $\mathcal{F}(E'') = \mathcal{F}(E)$ , there exist some set  $sus$  and some  $g_0$  such that  $\mathcal{F}(E) \subseteq sus$  and, for any  $p_i \in \mathcal{C}(E)$  and  $g$  ( $g \geq g_0$ ),  $sus_i^{E'',g} = sus$  hold. Since no processor executes a reset step in  $E''$ ,  $|sus| \leq n - k$  holds. Let  $Active$  be the value of  $pick_k(\{id_1, id_2, \dots, id_n\} - sus)$ . Then,  $|Active| = k$  and  $Active \subseteq \{id_j | p_j \in \mathcal{C}(E)\}$  hold. For any  $p_i \in \mathcal{C}(E)$  and any  $g \geq g_0$ ,  $sus_i^{E'',g} = sus$  holds, and therefore, and  $Active_i^{E'',g} = Active$  holds. Since  $E''$  is a suffix of  $E$ , this implies that  $E$  is a legal execution for the  $k$ -group consensus problem.  $\square$

## 5. Conclusion

We considered fault-tolerant and self-stabilizing protocols using an unreliable failure detector. We defined  $k$ -accuracy of the failure detector, and showed the  $k$ -accuracy is necessary for *ftss* protocols for the  $k$ -group consensus problem. We also presented three  $(n - k)$ -*ftss*  $k$ -group consensus protocols using the  $k$ -accurate and weakly complete failure detector, (1) a space-unbounded protocol on complete networks under the R/W daemon, (2) a space-unbounded protocol on  $(n - k + 1)$ -connected networks under the R/W daemon, and (3) a space-bounded protocol on complete networks under the C daemon. The first protocol shows that  $(n - k)$ -*ftss*  $k$ -group consensus can be solved in complete networks using a  $k$ -accurate and weakly complete failure detector even under the R/W daemon. We modified the protocol to the second one so that it should solve the problem in  $(n - k + 1)$ -connected networks, a larger class of networks than the first one. However, these two protocols are space-unbounded. We resolved this disadvantages for the C daemon, which assumes larger atomic actions than the R/W daemon. Though the third protocol achieves space-bounded under the C daemon, it can be applied only to complete networks. The space-boundedness is an important requirement for self-stabilizing protocols. Practically, we can prepare sufficiently large spaces for unbounded variables if they are used in some non-self-stabilizing protocol. However, self-stabilizing protocols and *ftss* protocols can be started from any configuration, and therefore, we cannot prepare a sufficiently large space for unbounded variables in advance. It is one of our future works to investigate the possibility of a space-bounded *ftss*  $k$ -group consensus protocol under the R/W daemon.

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